Stellar temperatures by Wien's law: Not so simple

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A star's surface temperature is among the most important features that can be deduced from its light. We have made measurements to see how reliably we could determine the surface temperatures of some A to K stars using Wien's Displacement Law. We took spectra, corrected them for atmospheric extinction and instrumental response, found the wavelengths of their intensity maxima, and then from Wien's law found the surface temperatures of the observed stars. For F to early K stars, our results agree with temperatures determined in other ways. For A and later K stars, the agreement is poor because the spectra are appreciably different from ideal blackbody spectra and because our equipment responds poorly to the deep red and blue wavelengths where the spectra of these stars have their peak intensities. This paper points out several interesting concepts in and outside the astrophysical domain that can be instructive for undergraduate students. © 2012 American Association of Physics Teachers.
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I. INTRODUCTION

We use a spectrograph especially built for taking the spectra of stars to find their surface temperatures. Such temperatures can be measured in various ways. In the present paper, we use a method based on Wien's displacement law. This method turns out to be not so simple as textbooks make it seem.

In this paper, we show how to analyze stellar spectra with basic equipment. We use our results to test the accuracy of Wien's law for inferring stellar surface temperatures. We discuss mainly the technical aspects and the underlying physics of our measurements rather than their pedagogical implications. However, the project we describe is well within the reach of undergraduate students.

Our paper is organized as follows: In Sec. II, we outline the main techniques for measuring stellar temperatures; in Sec. III, we describe our equipment, the spectra, and the data reduction procedure; in Sec. IV, we analyze the spectra and determine the temperatures; in Sec. V, we discuss our results; and in Sec. VI, we present some conclusions.

II. FINDING STELLAR TEMPERATURES: A SURVEY

Stellar temperatures are usually determined from spectra. The relative brightness at different wavelengths depends on temperature in a way that obeys the blackbody laws. Of course, stars are not perfect blackbodies and this is an issue we address in this paper. However, if the blackbody approximation holds, the temperature can be found by measuring the spectral distribution of the stellar light and matching it to an ideal blackbody curve. In principle, instead of measuring the entire spectral distribution, it is enough to find the wavelength of maximum intensity λ_{max} and then use Wien's displacement law. Wien's law relates λ_{max} to the blackbody's temperature T as

$$T = \frac{b}{\lambda_{max}}, \tag{1}$$

where $b = 2.898 \times 10^7$ K Å is Wien's constant.

Because the visible part of the spectrum extends from about 3800–7500 Å, we expect a star's spectral intensity to peak in the visible when its temperature is between 3850 and 7650 K.[1] The spectra of hotter stars will peak in the UV; those of cooler stars will peak in the IR. Because our observations are in the visible, we can only use Wien's law for intermediate-temperature stars. In addition, our CCD camera imposed a further limitation. Because it is capable of spanning only about 3000 Å across the frame, we only took data in the region between 3900 and 6900 Å. Thus, our target stars are limited to those with temperatures between 4200 and 7450 K (F, G, and early K stars).[2]

In practice, temperatures are not usually determined using Wien's law. Instead one measures the *color index*—the ratio of brightnesses of two wavelength intervals in the stellar spectrum. Such a ratio is enough to specify a Planck blackbody curve and thus determine a value for T.

There are pros and cons for each method. If stars were perfect blackbodies the two methods would be equivalent. But stars are not perfect blackbodies mainly because of the presence of absorption lines. The presence of a spectral line in the region of a spectrum's maximum brightness will shift the wavelength of the maximum and thus modify the value of T inferred from Wien's law. The color index method, on the other hand, is less sensitive to a single spectral line because it averages the flux over a wavelength interval of some width. In other words, it uses more of the information provided by the star. However, temperatures found by this method are sensitive to deviations from blackbody emissions that extend over larger spectral regions. One nice feature of the color index method is that it doesn't require observation of an entire stellar spectrum because the color index can be worked out simply

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from the measurements of stellar magnitudes with B and V (or other) filters.[3] Thus, you can use this method even if you do not own a spectroscope. A star's surface temperature, when measured using either Wien's law or the color index technique, is called its *color temperature*.

Another method of finding stellar temperatures is to use the Stefan–Boltzmann law

$$L = 4\pi R^2 \sigma T^4, \tag{2}$$

where $\sigma = 5.670 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4} is the Stefan–Boltzmann constant. This formula relates a star's temperature to the luminosity L and radius R of the star. If L and R are known, T can be derived from Eq. (2) and is called the *effective temperature*. The effective temperature is a more physically sound concept than color temperature because, being defined in terms of the stellar luminosity, it averages over all deviations from a blackbody spectrum. In other words, the definition does not depend on the chosen spectral region or the locations of single spectral lines. Unfortunately, with the exception of our Sun and eclipsing binaries, stellar radii are generally not known. Such radii can be measured for many stars, but only with complex interferometric techniques, so the effective temperature is not typically the simplest way to proceed.

Yet another way to determine stellar temperatures is to analyze the spectral lines themselves. As explained in Ref. 4, such lines are functions of temperature and this relationship can be exploited to find T.

In this paper, we describe a method based on Wien's law. Many astronomy textbooks introduce Wien's law as the first choice for students to understand the relationship between a star's spectrum and its temperature. We will see that using Wien's law is not as straightforward as textbook presentations lead students to expect.

For reference data to compare to our spectra, we use "A library of stellar spectra"[5] to find tabulated spectra that matched ours in spectral resolution and overall quality.

III. EQUIPMENT AND THE OBSERVATIONS

We use an SGS spectrograph coupled with an ST-7XE CCD camera (dynamical range = 16 bit, full well capacity ≈50 000 e$^-$), both manufactured by SBIG. The equipment can produce a spectrum that has an average dispersion of 4 Å/pixel. This instrumentation belongs to the astronomical dome of the "Liceo Ginnasio G. Parini" and is jointly used by the Liceo and the Dipartimento di Fisica (formerly Istituto di Fisica Generale Applicata) of Milan University to offer high school and university students the opportunity to perform astrophysical observations and measurements. Since the location within a large metropolitan area is unsuitable for some measurements, we developed a collaboration with the Osservatorio Astronomico della Regione Autonoma Valle d'Aosta (OAVdA) in the Italian Alps at 1650 m above sea level where the sky conditions are much better. At OAVdA, the spectrograph and CCD were mounted on a Cassegrain 250 mm f/10 telescope.

At OAVdA, we took some spectra under very good (photometric) conditions. From these, we selected for the present work spectra of a few stars of different spectral classes between F and early K. These stars are Procyon (F5), Capella (G8 + G0), Dubhe (K0), Arcturus (K1,5), and Albireo A (K3).[6] We also took a spectrum of an A star (Vega) because during our measurements it was shining gloriously overhead. Exposure times ranged from 5 to 20 s, enough to give reasonably good S/N ratios (around 50–100).

The spectra were dark-subtracted and calibrated in wavelength. To account for the non-linearity of the spectrum across the frame, we used several lines from emission lamps and fitted a curve through them. We used the Hα, Hβ, Hγ and Hδ lines; the He lines at 3964.73, 4471.48, 5015.68, 5875.62, and 6678.15 Å; and the Hg line at 5460.75 Å. The dispersion function turns out to be almost, but not exactly, linear.

It was also necessary to correct the spectra for atmospheric reddening[7] and for changes in the incident spectra produced by the spectral response of the combined telescope + spectrograph + CCD system. These corrections are described in what follows.

A. Atmospheric reddening

To correct for atmospheric reddening we used a simple model based on Rayleigh scattering in the atmosphere. As we didn't observe stars close to the horizon, we can neglect the Earth's curvature and represent the atmosphere as in Fig. 1. Let P be the observation site, and r and z the distances from point P along the line of observation and in the vertical direction, respectively. Letting θ be the angle between these two directions (the zenith angle), we have

$$z = r\cos\theta. \tag{3}$$

Now, let $N(z)$ be the number of molecules per unit volume (number density) in the atmosphere at altitude z. The density profile for increasing z can be expressed as

$$N(z) = N_0 e^{-z/z_0} \Rightarrow N(r) = N_0 e^{-r/r_0}, \tag{4}$$

where N_0 is the number of particles per unit volume at sea level, $r_0 = z_0/\cos\theta$, and $z_0 = 7640$ m is the scale height of the atmosphere.[8] We can calculate N_0 in the following way. The air density at sea level is $\rho_0 \approx 1.2 \times 10^{-3}$ g cm^{-3}, and if we treat air as a single component diatomic gas with the atmosphere's average molecular weight $\mu = 28.99$ amu, then

$$N_0 = \frac{\rho_0 N_{AV}}{\mu} = 2.49 \times 10^{25}\ \text{m}^{-3}, \tag{5}$$

where N_{AV} is Avogadro's number.

Next, consider a given stellar spectrum. Let $I(\lambda, \infty)$ be the spectral flux—the energy per unit time, per unit area, and per unit wavelength—at the top of Earth's atmosphere. This is the actual spectral distribution emitted by the star. The

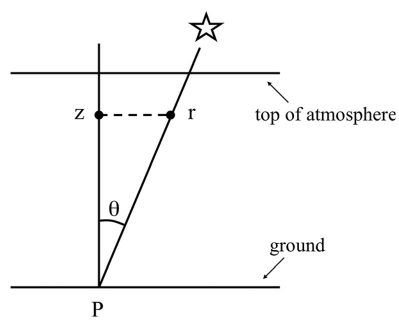

Fig. 1. Path of the stellar light across the terrestrial atmosphere; r and z are the distances from some point P along the line of observation and in the vertical direction, respectively; they increase upwards.

symbol "∞" means that this is the spectral flux for $r \to \infty$ (outside the atmosphere) before any extinction (e.g., scattering and absorption) processes have occurred. As light passes through the atmosphere, there is some extinction and the spectrum $I(\lambda, \infty)$ becomes the function $I(\lambda, r)$ to be determined. The extinction through a slab of thickness dr can be expressed by

$$dI(\lambda, r) = I(\lambda, r) c(\lambda, r)\, dr. \tag{6}$$

Notice that in Eq. (6), both $dI(r)$ and dr are negative for light moving downwards from the top of the atmosphere, so that $dI(r)/dr > 0$. The term $c(\lambda, r)$ describes atmospheric extinction as a function of both λ and r. The λ-dependence is due to the optical properties of the atmosphere, while the r-dependence arises because the air density profile decreases the spectral flux. We can suppose for simplicity that this decrease is proportional to $N(r)$. Then, we can write $c(\lambda, r)$ as the product of two separate factors, one depending only on λ and the other only on r:

$$\begin{aligned} c(\lambda, r) &= N(r)\sigma(\lambda) = N_0 \mathrm{e}^{-r/r_0}\sigma(\lambda) \Rightarrow dI(\lambda, r) \\ &= I(\lambda, r) N_0 \mathrm{e}^{-r/r_0}\sigma(\lambda) dr, \end{aligned} \tag{7}$$

where $\sigma(\lambda)$ is the extinction cross section (for both absorption and scattering).

To estimate this cross section, assume the observations were made under ideal conditions with no aerosols or dust present (photometric nights). Absorption by air molecules throughout the visible is practically negligible (up to Fraunhofer's B band beyond 6800 Å that is due to telluric oxygen). Under these ideal conditions, extinction can be assumed to be due only to Rayleigh scattering.

The cross section for Rayleigh scattering is related to the polarizability $\alpha(\lambda)$ of each molecule of air through the relation

$$\sigma(\lambda) = \frac{128\pi^5}{3} \frac{1}{\lambda^4} \alpha(\lambda)^2, \tag{8}$$

and $\alpha(\lambda)$, in turn, is given by the Lorentz–Lorenz formula[9]

$$4\pi\alpha(\lambda) N(r) = 3\frac{n^2 - 1}{n^2 + 2}, \tag{9}$$

where n is the refractive index of air (which holds for the entire atmosphere). Notice that α depends only on λ, N depends only on r, and n depends on both.

Solving Eq. (9) for α and substituting into Eq. (8) gives

$$\begin{aligned} \sigma(\lambda) &= 24\pi^3 \frac{1}{N(r)^2} \frac{1}{\lambda^4} \left(\frac{n^2 - 1}{n^2 + 2}\right)^2 \\ &= 24\pi^3 \frac{1}{N(r)^2} \frac{1}{\lambda^4} \frac{(n-1)^2 (n+1)^2}{(n+2)^2}. \end{aligned} \tag{10}$$

As the refractive index of air is equal to $1 + \varepsilon$ with $\varepsilon \ll 1$, we can assume that $n+1 \approx 2$ and $n+2 \approx 3$ in Eq. (10); no approximation is allowed for $n-1$ because it is equal to ε itself. In other words, the quantity ε is additive in $n+1$ and $n+2$, so it can be neglected, but in $n-1$ it multiplies the entire equation and plays a major role. We can, therefore, write

$$\sigma(\lambda) = \frac{32\pi^3}{3} \frac{1}{N(r)^2} \frac{1}{\lambda^4} (n-1)^2. \tag{11}$$

To evaluate the molecular scattering cross section $\sigma(\lambda)$, we use the values for $N(r)$ and $n-1$ at the sea level to get

$$\sigma(\lambda) = \frac{32\pi^3}{3} \frac{1}{N_0^2} \frac{1}{\lambda^4} (n_0 - 1)^2. \tag{12}$$

To estimate n_0 we use the Cauchy relation[10]

$$n_0 - 1 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}, \tag{13}$$

where (for pure air at STP) $A = 0.000287566$, $B = 1.3412 \times 10^{-18}\ \mathrm{m}^2$, $C = 3.777 \times 10^{-32}\ \mathrm{m}^4$, giving the final result

$$\sigma(\lambda) = \frac{32\pi^3}{3} \frac{1}{N_0^2} \frac{1}{\lambda^4} \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}\right)^2. \tag{14}$$

It is clear from Eq. (14) that the celebrated dependence on λ^{-4} of the Rayleigh scattering cross section is not exactly true. On the other hand, it is not such a bad approximation either. In fact, looking at the values of the coefficients A, B, and C, it is clear that $A \gg B\lambda^{-2}$ and $A \gg C\lambda^{-4}$ if $\lambda \gg 10^{-7}$ m $= 10^3$ Å, so that in the visible the whole term in parentheses changes only slightly with λ.

Integrate through the atmosphere, Eq. (7) becomes

$$\frac{dI(\lambda, r)}{I(\lambda, r)} = \frac{32\pi^3}{3N_0} \frac{1}{\lambda^4} \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}\right)^2 \mathrm{e}^{-r/r_0} dr, \tag{15}$$

which is then integrated over r to give

$$\int_{r_1}^{\infty} \frac{dI(\lambda, r)}{I(\lambda, r)} = \frac{32\pi^3}{3N_0} \frac{1}{\lambda^4} \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}\right)^2 \int_{r_1}^{\infty} \mathrm{e}^{-r/r_0} dr. \tag{16}$$

Integration from r_1, which corresponds to the height z_1 of the observation site, to ∞, which is the top of the atmosphere, yields

$$\ln \frac{I(\lambda, \infty)}{I(\lambda, r_1)} = \frac{r_0}{\mathrm{e}^{r_1/r_0}} \frac{32\pi^3}{3N_0} \frac{1}{\lambda^4} \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}\right)^2. \tag{17}$$

As expected, the right hand side is positive, which means that $I(\lambda, \infty) > I(\lambda, r_1)$ at any given λ. Equation (17) immediately results in

$$\frac{I(\lambda, \infty)}{I(\lambda, r_1)} = \exp\left[\frac{r_0}{\mathrm{e}^{r_1/r_0}} \frac{32\pi^3}{3N_0} \frac{1}{\lambda^4} \left(A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}\right)^2\right], \tag{18}$$

which gives, as a function of wavelength, the ratio of the spectrum at the top of the atmosphere to that actually collected at the entrance pupil of the telescope. Plugging in for r_0 and r_1 (the height above sea level of OAVdA is $z_1 = 1650$ m) we then have

$$I(\lambda,\infty) = I(\lambda,r_1)\times\exp\left[8.18\times10^{-20}\frac{1}{\cos\theta}\frac{1}{\lambda^4}\left(A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}\right)^2\right]\approx I(\lambda,r_1)\left[1+8.18\times10^{-20}\frac{1}{\cos\theta}\frac{1}{\lambda^4}\left(A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}\right)^2\right]. \quad (19)$$

As a test, we can insert $\lambda = 4\times10^{-7}$ m (blue) and $\lambda = 6.5\times10^{-7}$ m (red), and find that for a zenith angle of 0, Eq. (19) gives corresponding absorptions of around 30% and 5%, respectively; these are reasonable results. In addition, our approach has the advantage that it allows us to calculate absorption for any height above the horizon and for any value of λ, not just some particular wavelengths. It also shows the physical processes involved in atmospheric extinction.

B. Instrumental response

The spectral flux $I_1(\lambda, r_1)$ recorded at the output of the telescope + spectrograph + CCD system is different from the true spectral flux $I(\lambda, r_1)$ arriving at the telescope input. The differences are caused by the telescope and associated apparatus. In correcting for these instrument effects, it is convenient to consider the telescope + spectrograph + CCD as a whole. Let the quantum efficiency $QE(\lambda)$ be the detection efficiency per given number of incident photons of the whole instrument (and not, as usual, of the CCD alone) as a function of wavelength

$$QE(\lambda) = I_1(\lambda,r_1)/I(\lambda,r_1). \quad (20)$$

To get a reliable measurement of QE we used a light source with a known emission spectrum and compared it with the spectrum collected by our equipment. Because we did not have available a calibrated laboratory source, we used a star whose spectrum is tabulated in Ref. 5. We took a spectrum of this star with our CCD, corrected it for atmospheric reddening, and compared it with the tabulated spectrum.

Notice that in evaluating the QE it is not important in which order atmospheric reddening and instrumental response are corrected. Both corrections—the term within parentheses in Eq. (19) and the reciprocal of the QE—multiply the spectrum. Hence, we can de-redden the observed spectrum and then divide it by the tabulated spectrum to obtain the QE.

Corrections for vignetting and varying sensitivity across the frame have been included in the measurement of the QE. This procedure is acceptable only if the same wavelengths always fall on the same pixels when taking different stellar spectra. Although we tried to adhere strictly to this condition, very small shifts in the equipment can occur when observing different spectra. We notice, in any case, that a 10–20 Å shift has a minor bearing on the results, because it only results in a 10–20 K error. As we will point out in Sec. V, we do not expect a precision higher than 50 K.

A G-class star is a good calibration star because it has good emission throughout the whole visible spectrum. We chose Kornephoros (β Her), a star classified G7III.[11] We looked for a G7III spectrum in Ref. 5, and we calculated the QE as explained earlier, performing a sampling in wavelength each 100 Å throughout the whole spectral range of interest from 3900 to 6900 Å. For simplicity's sake, we normalized the QE to have a maximum value of 1.

We found that our equipment has poor sensitivity in the blue. This insensitivity, together with atmospheric reddening and the small aperture of our telescope, kept us from obtaining high S/N ratios in the blue spectral region. As a result, we suspected the presence of artifacts in our spectra around 4000 Å, and we decided to consider only wavelengths greater than 4200 Å. Although we didn't expect our spectra to be very good from 4200 to 4500 Å, the slight increase both in the QE and in the brightness of the calibration star made us confident that our results in this spectral region are reliable enough to be useful.

We calculated the points of the graph of the QE function shown in Fig. 2 from data taken every 100 Å and then connected them by eye with a smooth curve (thin line). Subsequently, we fit the data with a cubic polynomial

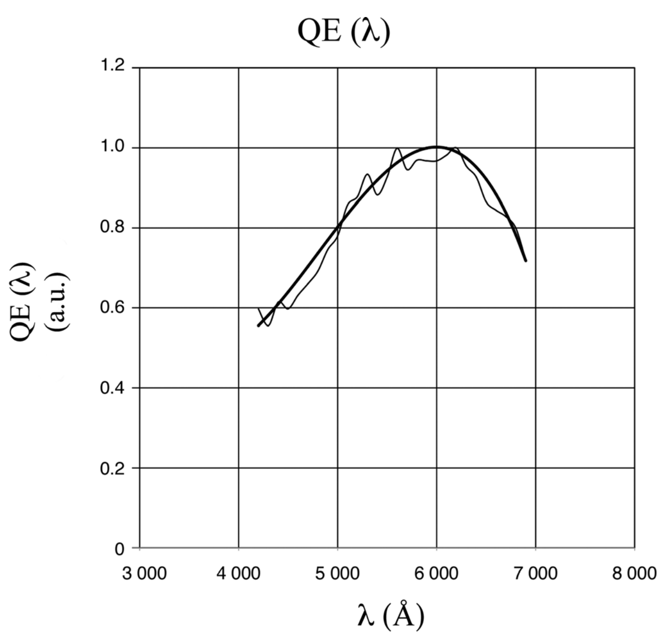

Fig. 2. Plot of $QE(\lambda)$ normalized to its maximum value. The thin line is the QE worked out from the calibration star (empirical curve); the bold line is a polynomial fit [Eq. (21)].

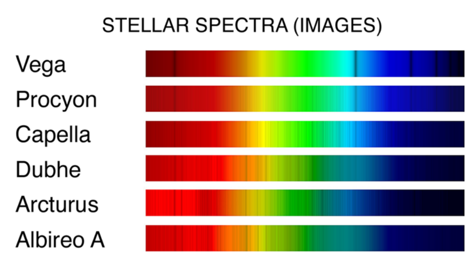

Fig. 3. (Color online) Spectra of the stars in our sample. These spectra are uncorrected both for atmospheric and instrumental reddening. The colors are inserted by software once the wavelength calibration is carried out, and they are only indicative. They are interesting nonetheless, because they can be used to attempt a classification in the Harvard scheme.

$$QE(\lambda) = -7.667 \cdot 10^{-11}\lambda^3 + 1.106 \cdot 10^{-6}\lambda^2 - 4.993 \cdot 10^{-3}\lambda + 7.687 \quad (21)$$

to have a convenient way to interpolate between the calculated values.[12] The result is shown by the bold line in Fig. 2.

The differences between the empirical and calculated curves are significant. They occur because, instead of a laboratory blackbody source, we calibrated using a star that has its own spectral lines. In other words, the complicated structure of the empirical curve reflects the complex shape of the calibration spectrum. It is clear that this method may introduce some degree of imprecision that must be considered when discussing errors (Sec. V).

Is it worthwhile to develop a detailed model for atmospheric extinction when such residuals are present in our

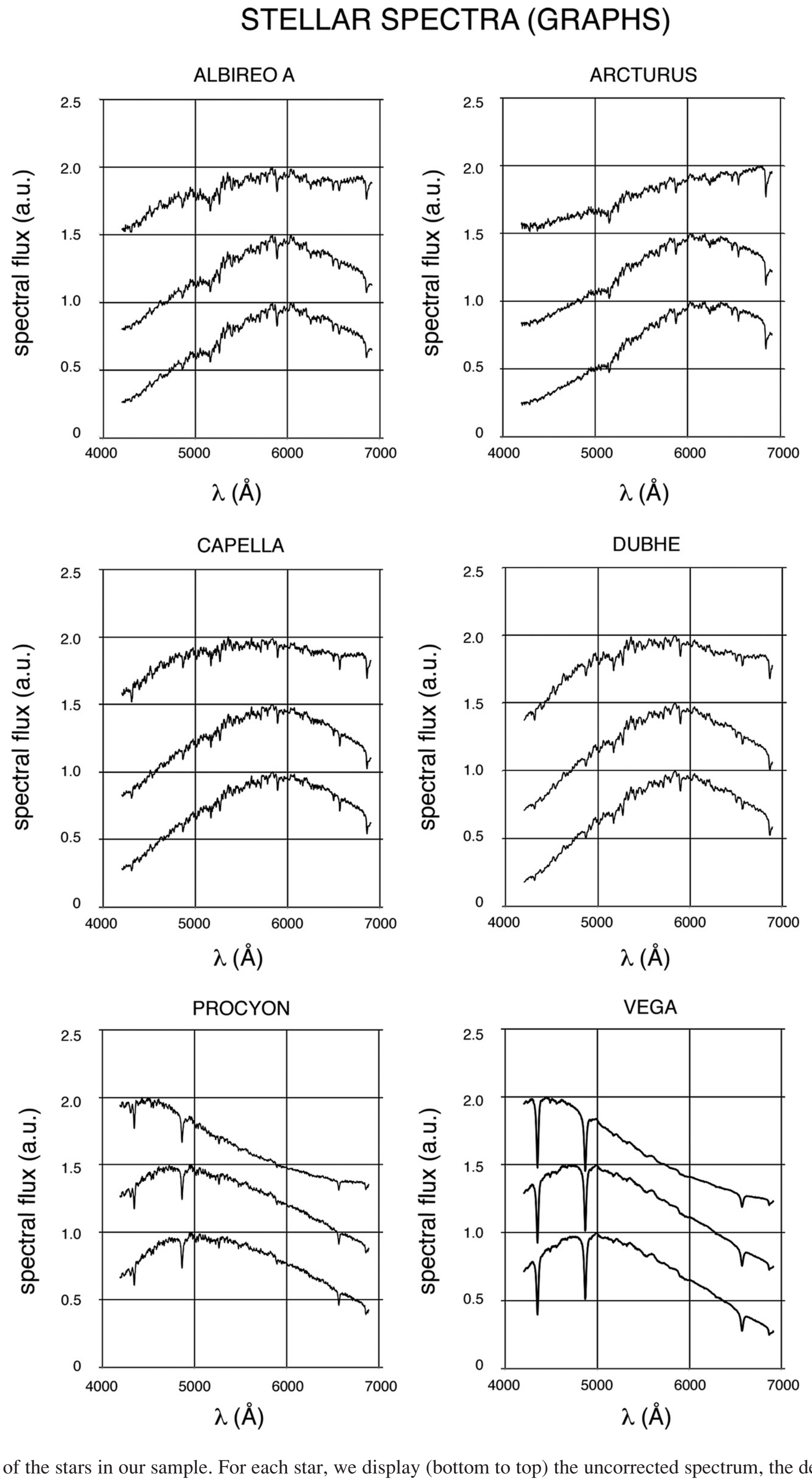

Fig. 4. Spectral distributions of the stars in our sample. For each star, we display (bottom to top) the uncorrected spectrum, the de-reddened spectrum, and the spectrum corrected also for instrumental response. The spectra have been normalized by putting their maximum value equal to 1 and have been offset vertically by 0.5 units.

Table I. Estimate of stellar surface temperature in our spectra. The final result of our measurements is given by T_2 (bold).

Star	Spectral class	T	λ_1	T_1	E_1	λ_2	$\boldsymbol{T_2}$	E_2
Albireo A	K3 II	4100	6050	4800	+17.1	6050	**4800**	+17.1
Arcturus	K1.5 III	4300	6200	4650	+8.1	6750	**4300**	0.0
Capella	G8 III + G0 III	5300	5850	4950	−6.6	5350	**5400**	+1.9
Dubhe	K0 III	4500	5850	4950	+10.0	5850	**4950**	+10.0
Procyon	F5 IV-V	6550	4950	5850	−10.7	4450	**6500**	−0.8
Vega	A0 V	9500	5000	5800	−38.9	4450	**6500**	−31.6

evaluation of the QE? We think it is because (1) a full treatment of atmospheric reddening has pedagogical value, and (2) the use of a calibration star for the calculation of the QE is, in the absence of a laboratory calibration lamp, the right line of attack even if it results in reduced accuracy of the instrument's reddening correction.

IV. ANALYSIS OF OUR SPECTRA

The spectra of the six stars of our sample were wavelength-calibrated and then stored both as images (TIFF files) and as text files in which along with each pixel there is associated a wavelength and a photon count. Images of the spectra are shown in Fig. 3.

We divided the photon count by λ to transform it into energy (to within some constant factor). This procedure gave us each star's spectrum $I_1(\lambda, r_1)$ that we then corrected for atmospheric and instrumental reddening as described above.

Graphs of these spectral distributions are shown in Fig. 4. Note that atmospheric de-reddening is not nearly as important as the correction for the instrument response.

Table I shows the stellar temperatures that we determined. For each star, the table shows the spectral class and surface temperature T,[13] the wavelength λ_1 at which the uncorrected spectrum peaks, the wavelength λ_2 at which the corrected spectrum peaks, and the color temperatures T_1 and T_2 that we deduced by applying Wien's law to λ_1 and λ_2, respectively. In Table I, E_1 and E_2 are, respectively, the percentage differences of T_1 and T_2 relative to T.

In working out λ_1 and λ_2, we have neglected the absorption lines that produce deviations from the blackbody law. We would like to figure out the position of the maximum as if absorption lines were not present, but as can be seen from the magnified parts of the spectra of Procyon and Capella shown in Fig. 5, this is not likely to be an easy task.

V. DISCUSSION

The straightforward application of Wien's law to stellar spectra in the range of wavelengths where our equipment is responsive gives reasonably good results. Of the six stars we report here, four have temperatures within roughly 10% of the expected value; this is acceptable agreement. Because of the limitations of our equipment, we did not expect good agreement for Vega, but the result for Albireo is surprisingly different from what we expected from Wien's law. Such discrepancies occur for similar spectra tabulated in Ref. 5 (although to a lesser extent).[14]

There are two main reasons for this. First, there are errors and approximations in our treatment. And second, stars are not perfect blackbodies. We will briefly discuss both these issues.

A. Errors and approximations

The sources of errors and approximations can be summarized as follows:

(1) We assumed "perfect" atmospheric conditions and tried to use only observations made on very clear nights. However, we cannot thoroughly exclude that there were small deviations from such conditions;
(2) We carried out a sampling and an interpolation to estimate the QE, which resulted in sizeable residuals because we used a star as a calibrator;
(3) We relied on the assumption that no shift occurred when passing from one spectrum to another. This is not strictly true, although we estimated that such shifts were acceptable;
(4) We used our CCD camera up to the edges of its frame. We cannot expect to have very good response in these regions, so our spectra are less reliable at the shortest and longest wavelengths we considered (this point is discussed further in Sec. V B);

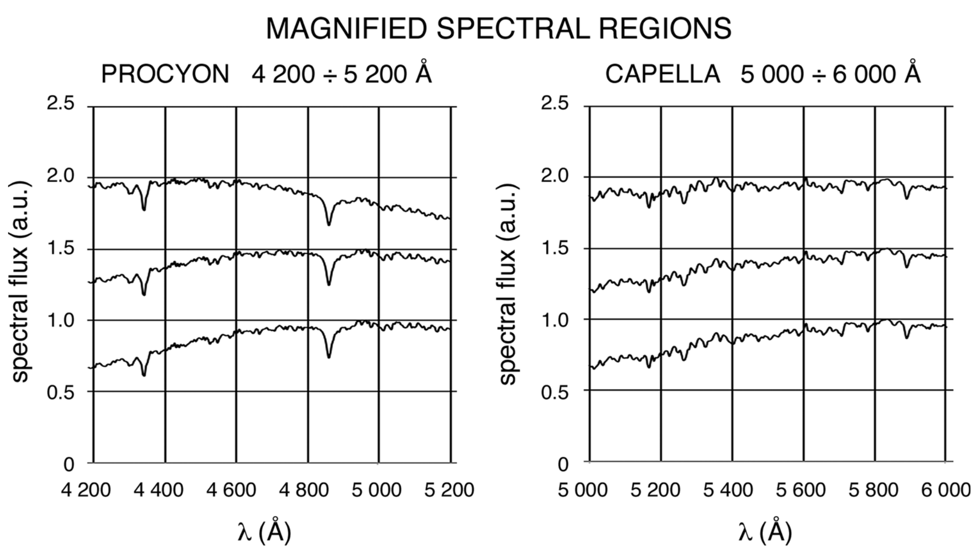

Fig. 5. Magnified regions in Procyon's and Capella's spectra. The three graphs have the same meaning as in Fig. 4.

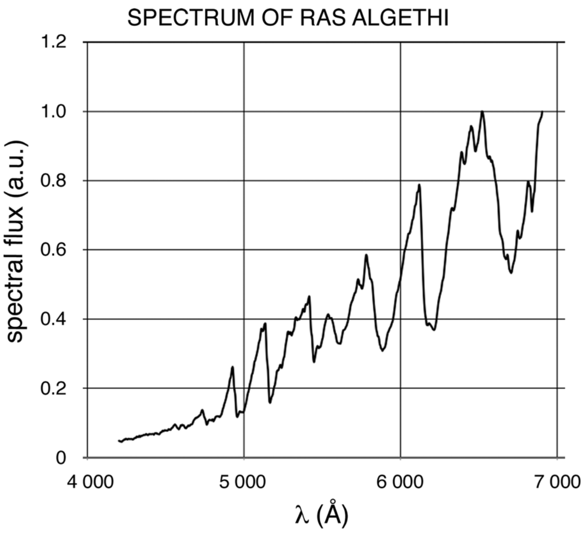

Fig. 6. Spectrum of Ras Algethi, an M5II star.

(5) For Albireo, there might be a slight degree of interstellar reddening to consider.

We emphasize that points (2) and (4) are the most important, while (5) is practically negligible as compared to the others. All things considered, we estimate a best case precision of around 50 K.

B. Stars are not blackbodies

Finally, stars are not blackbodies. The presence of absorption lines introduces deviations from a perfect blackbody spectrum. These deviations can be more or less significant depending on spectral class, and they are not easy to account for. Generally speaking, intermediate classes—typically late F to early K—behave better in this respect because such stars have many metallic lines that are generally narrow and don't affect the overall shape of the spectrum very much.

Things are quite different outside this interval. Consider the hotter A to early F stars (O and B stars were ruled out from the beginning). These stars, at least in the luminosity classes III to V, develop deep, strong-winged Balmer lines that cause an abrupt fall in brightness below 4000–4200 Å where the wings of these lines begin to overlap heavily. The result is that the spectra of these stars all reach a maximum around these wavelengths irrespective of the star's actual temperature. Wien's law would give a temperature of 7000–7500 K, while the actual temperatures range from roughly 7000–12000 K. The upper limit of this interval coincides with the onset of hydrogen ionization that makes the Balmer lines disappear and restores a blackbody shape in stellar spectra but with a peak in the ultraviolet. Other techniques must be used for these stars, like careful calibration of color indices or analysis of the features of the Balmer Jump.

Let's now consider cooler K and M stars. Their spectra begin to develop a large number of lines and absorption bands that overlap throughout large spectral regions and create significant deviations from blackbody curves. For example, Fig. 6 shows a spectrum we took of Ras Algethi, an M5II star. As is typical for such very late spectral types, although the trend of increasing flux towards longer wavelengths is clear throughout the visible, it is difficult to discern an actual "continuum" among the overlapping lines and bands. In any case, the peak shifts to the infrared.

Table II. Classifications of our spectra compared with tabulated classifications.

Star	Tabulated spectral class	Our estimate
Albireo A	K3	K2
Arcturus	K1.5	K4
Capella	G8 + G0	G8
Dubhe	K0	K0
Procyon	F5	F7
Vega	A0	A5

Among the stars we observed, we found good results for Procyon, Capella, Dubhe, and Arcturus. On the contrary, Vega's value is quite incorrect as was to be expected from our earlier discussion. In Albireo's case, we hoped for better results. We expected the peak to shift towards the 6500 Å region, but the maximum remained around 6050 Å both before and after correcting the spectrum. Although the red portion increased in brightness, this was not enough to shift the maximum to longer wavelengths. This result is possibly due to the relatively poor response at the extreme borders of the CCD, which is an important issue. Such poor response would also be a problem for high precision spectrophotometric measures. In the present case, we can accept this imprecision provided that we know it exists and we do not expect to have a very precise measure of spectral flux at the extreme border of the CCD (towards 6800–6900 Å).

We also notice that in the case of Dubhe, the peak was unchanged before and after correcting the spectrum. However, this behavior was expected because of the presence of a prominent increase in spectral flux around 5850 Å. From Ref. 5, we can see that a K0III spectrum like Dubhe's actually peaks in the 5800–6000 Å region, as we found.

Given that Wien's law doesn't apply to many stellar spectra, we can try to classify our spectra in the Harvard scheme by comparing them with spectra tabulated in Ref. 5. This means trying to identify which spectra of Ref. 5 most resemble ours in overall appearance. Although this approach has limited value because it is not based on precise measurements, we mention it as a possible exercise to help acquaint students with the shape of stellar spectra. The results of our own estimates are reasonably good and are shown in Table II.

VI. CONCLUSIONS

We were curious to see if the often-cited application of Wien's law to stellar spectra can actually provide good results. We found that it does in some cases but not in others, particularly when the stars are not perfect blackbodies.

In addition to exploring the precision with which stellar temperatures can be determined using Wien's law, our work illustrates how to reduce and analyze stellar spectra, and it provides an occasion to examine the meaning of "stellar temperature." Moreover, our investigation casts light upon important concepts in and outside of astrophysics, and we think it could be a suitable astrophysics laboratory project for undergraduate students.

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[1]Here and in the following we approximate all temperatures to the nearest 50 K (see Sec. V for an explanation of this choice).

[2]Stellar spectra are usually classified according to a so-called Spectral Classification (Harvard scheme): there are seven main spectral classes, labelled by the capital letters O, B, A, F, G, K and M and ordered from higher to lower surface temperature. O stars are the hottest, with temperatures up to 30 000 K or higher, and M stars are the coldest, down to 3000 K; F and G stars have intermediate temperatures with values around 6000–7000 K. Each class is further divided into ten types labeled by the numbers 0, 1, 2, … , 9. Once astronomers thought that stars evolved cooling down, and they spoke of "early" (hotter) or "late" (colder) spectral classes. Even within one class, they spoke of "early" and "late" types: for example K1 is an early K-type, and K8 a late one. Such ideas about evolution were discarded decades ago, but this terminology is still used. Moreover, for a given temperature, luminosity is related to radius, and in the subsequent MK classification scheme stars were given a "luminosity class" (indicated by a roman numeral I, II, etc.) according to their luminosity or dimension. To wit, class I stars are the largest, or supergiants, while class V stars are the much smaller main sequence stars. For elaboration on this point, see, *Fundamental Astronomy* edited by H. Karttunen, P. Kröger, H. Oja, M. Poutanen, and K. J. Donner, 3rd ed. (Springer, Berlin/Heidelberg/New York, 1996) pp. 236–240.

[3]About the concept of stellar magnitude, see Karttunen *et al*. (1996) cited in Ref. 2, pp. 97–101. The B and V filters are filters centered on the blue (B) and yellow-green (V for *visible*) part of the spectrum.

[4]The paper is D. Cenadelli and M. Zeni, "Measuring stellar temperatures: An astrophysical laboratory for undergraduate students," Eur. J. Phys. **29**, 113–121 (2008). The method is based upon the concept of equivalent width, for which see also R. C. Smith, *Observational Astrophysics* (Cambridge U.P., Cambridge, 1995), p. 182, or K. Robinson, *Spectroscopy: The Key to the Stars* (Springer, London, 2007), pp. 52–57.

[5]G. H. Jacoby, D. A. Hunter, and C. A. Christian, "A library of stellar spectra," Astrophys. J., Suppl. Ser. **56**, 257–281 (1984). Another reference book that gives full account of the science of stellar spectroscopy is R. O. Gray and C. J. Corbally, *Stellar Spectral Classification* (Princeton U.P., Princeton, 2009).

[6]Data about the stars (spectral classes and temperatures) here and in what follows are taken from Internet-available resources like the SIMBAD database or the site of Prof. James B. Kaler (Professor Emeritus of Astronomy, University of Illinois): <stars.astro.illinois.edu/sow/sowlist.html> (accessed June 2011). Capella is a well-known double; as both stars have similar spectra and brightness we kept it under consideration and when speaking of temperature, we mean the average temperature of the two stars.

[7]There also exists reddening due to interstellar matter. However, it only becomes significant for distances of more than a thousand light years or so, and the stars we observed are closer than a few hundred light years, so we can neglect interstellar reddening in our case.

[8]See J. M. Picone, A. E. Hedin, D. P. Drob and A. C. Aikin, "NRL-MSISE-00 Empirical model of the atmosphere: Statistical comparisons and scientific issues," J. Geophys. Res. **107**, 1468–1483, doi:10.1029/2002JA009430 (2002).

[9]J. D. Jackson, *Classical Electrodynamics* (New York, Wiley, 2000), p. 155.

[10]F. A. Jenkins and H. E. White, *Fundamentals of Optics*, 4th ed. (McGraw-Hill, New York, 1981).

[11]The star is located 148 light years away and interstellar reddening can be safely neglected.

[12]We carried out this interpolation via the website <www.arachnoid.com/polysolve/index.html> by Paul Lutus (accessed June 2011).

[13]All data are taken (as said in Ref. 6) from the SIMBAD database or the site of Prof. Kaler, except the temperature of Albireo A (HD 183915), that is taken from T. ten Brummelaar, B. D. Mason, H. A. McAlister, L. C. Roberts Jr., N. H. Turner, W. I. Hartkopf, and W. G. Bagnuolo Jr., "Binary star differential photometry using the adaptive optics system at Mount Wilson Observatory," Astron. J. **119**, 2403–2414 (2000).

[14]Not all the spectral classes of our stars are present in Ref. 1, but when this is the case, there is always a "very similar" spectrum we can resort to. This is the case for Albireo A (for which we utilized as a reference a K3III spectrum), for Arcturus (K2III) and Vega (A1V).

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